Aim
To introduce students to sequences on the calculator

Calculator objectives
By the end of this unit, you should be able to:

• generate a sequence recursively using the Calculator App.
• evaluate sequences, defined both as explicit formula and recurrence relations, at specific values
• plot sequences
• analyse a sequence using both the Function Table and a List & Spreadsheet (L&S) page

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Generating Sequences

A linear sequence of numbers, such as 2, 5, 8, … can be generated very easily in the Calculator App.

1. The sequence 2, 5, 8, … has an initial term 2. We then add 3 to get the next term.

   Type 2 then press [Enter].

   Press (this will paste an ‘Ans’) + 3.

2. The question is now ‘Ans + 3’. When you press press [Enter], this is evaluated as ‘2 + 3’, which returns 5.

   Pressing [Enter] again will re-evaluate the question (which is Ans + 3) as ‘5 + 3’, giving an answer of 8.

   This can be continued as many times as needed, thus generating a linear sequence.

More complex sequences can be generated in a similar way, through the use of ‘Ans’.
For example:

3. The sequence 4, 11, 32, can be generated by starting with 4, then multiplying the previous term by 3 and adding 1. This is done on the calculator as shown:

   ‘Ans’ is obtained by pressing (on) (on).

4. The sequence 5, 6, 13, 118, … can be generated by starting with 5. Subsequent terms are generated using the ‘formula’: $\text{Ans}^2 - 4\text{Ans} + 1$. 
Explicit Formulae

1. In a Calculator page, define your explicit formula: Press:
   
   [Menu], [1:Actions], [1:Define].

   and type \( u(n) = n^2 - 3 \)

2. Evaluate the explicit formula at various values of \( n \): Notice that:
   
   - before evaluating, the ‘u’ is in bold, to show that it is an assigned variable
   - the formula is defined for all values of \( n \) (including rational values)
**Recurrence relations**

Defining a recurrence relation in the Calculator App is slightly more complicated, as a piecewise function needs to be defined.

1. Define the recurrence relation, \( v \). Use the templates found on [CTRL]+[x] to set up the piecewise function. The initial condition must be in the first row of the piecewise function.

   ![Image of piecewise function setup](image1)

2. Defining a more complicated recurrence relation, e.g. the Lucas sequence, can be defined in the same way, with more rows in the piecewise definition. The initial terms must be in ascending order.

   ![Image of Lucas sequence definition](image2)
Plotting sequences

Open a new G&G page and change the Graph Type to Sequence:

[MENU], [3: GRAPH TYPE], [5: SEQUENCE]

In the formula entry bar, we now have space for the explicit formula or recurrence relation, initial terms for a recurrence relation, and the option to change values of \( n \) and the step size.

Plotting an explicit formula:

1. In the formula entry bar, enter \( u_1(n) = 0.2n + 3 \). The initial term, \( u_1(1) \) should be left blank (you will need to delete the 0 that is there by default), unless you want the initial term to have a value different from what the formula would give.

2. On pressing [ENTER], the formula entry bar will disappear. To bring it back, press [TAB], or [CTRL] + G.

3. To trace along the plot, choose Graph Trace:

   [MENU], [5:TRACE], [1:GRAPH TRACE].

Move left or right along the plot. Trace information is displayed in the bottom right corner as shown.
Plotting a recurrence relation:

1. The plot of a recurrence relation is defined in a similar way as before. Be careful to set up the recursive part of the relation using the same notation as the left-hand side of the equation, i.e. use $u_1(\cdot), u_2(\cdot)$, etc…

2. For more complicated recurrence relations, extra initial terms can be defined in order, i.e. $u_1(1), u_1(2)$, etc… For example, with the Lucas sequence:

3. The values of $n$ can also be changed from the default of $1 \leq n \leq 99$, by editing the 3rd row of the formula entry bar.

To avoid the calculator slowing down (spinning clock), consider reducing the size of the domain to, say, $1 \leq n \leq 10$.

The step size can also be changed to any value by editing the $n_{\text{step}}$.

What is the effect of changing the domain of the function? – horizontal translation.

4. Follow the same steps to trace the plot, as for explicit formula.
Exploring Sequences with Tables

There are two ways to view a table of values generated by a graph:

i. Using a L&S page
ii. Using a Function Table in a G&G page

Both types of sequences can use the function table. A recurrence relation that was originally defined in a G&G page can only use the function table.

Table in a L&S page

1. In a G&G page, define the sequence $u_1(n) = 0.5n + 3$.

2. Insert a new L&S page: [HOME], [3:Lists & Spreadsheets].

3. Give column A the name ‘n’, and in the formula cell of column B, enter $u_1(n)$. Remember to use the [var] key to choose the variable $n$.

4. Enter the value of the independent variable, $n$, in column A; the value of the dependant variable, $u_1(n)$ is automatically calculated in column B.
Function Table

1. In a G&G page, define the sequence \( u_1(n) = u_1(n) + 0.3 \).

2. Create a function table, either by pressing [CTRL] + T, or by pressing [MENU], [2:VIEW], [9: Add Function Table].

3. Scroll up or down the function table using the cursors.

4. To change the table set up, select: [Menu], [5:Function Table], [3:Edit Function Table Settings]

Reminder: Only sequences defined by an explicit formula in a G&G page can be analysed in a L&S page. A recurrence relation can be analysed in a L&S page if it is defined there.